A **ratio** compares two numbers in order. Ratios are written with a colon, like this:

4:3

The order of numbers in a ratio is important. The ratio 4:3 expresses a different relationship than the ratio 3:4 does.

The units of measurement that each of the two numbers expresses is important too. The two numbers in a ratio do not have to have the same units. One chicken to every pot (1 chicken:1 pot) is a ratio. Saying that one British Pound is worth about 1.5 US dollars (1 GBP:1.5 USD) is another ratio.

In some cases, it can be convenient to express ratios all in the same units, for example, the ratio of 9 cups to 16 cups may be written in two ways: with a colon 9:16 or as a common fraction 9/16. The fraction 9/16 is another way of saying 9 divided by 16.

The two numbers in a fraction are referred to so often that they have been given names. The top number in a fraction is the numerator and the bottom number is the denominator:

numerator / denominator

A fraction is another way of expressing one number divided by another. What is 24/56? You can take out a calculator and enter

24 / 56 = 0.42857

Incidentally, if you wanted to know what percent this fraction represented, simply multiply the number by 100, or move the decimal two places to the right

A **proportion** is similar to a ratio, except that it indicates a part of a whole, and so the numerator arises from the denominator. For instance, a researcher might say that, for every ten students in the residence hall, five were women. Five over ten (5 / 10) is a proportion. Proportions must have all of the numbers in the same units, and are frequently written as fractions.

What happens when you want to write a proportion, but the numbers are given in different units? Suppose you were asked to write the proportion of 3 cups to 56 ounces (3 cups out of 56 ounces). You must write a proportion of either cups to cups (cups:cups) or ounces to ounces (ounces:ounces), so you will have to convert one of the numbers so that both numbers are expressed in the same unit of measurement. Let's convert cups to ounces so we can express the ratio as ounces:ounces. Since there are 8 ounces in one cup, 3 cups are equal to 24 ounces:

3 cups * 8 ounces/cup = 24 ounces

Now the two numbers 3 cups and 56 ounces can be written as the following ratio:

24 ounces to 56 ounces,

24:56,

or, now that the unit of measurement is the same, 24:56 can also be written as a proportion:

24/56

You may need to solve some problems involving ratios.

If you divided 36 into two parts in the ratio of 1:2 and one part is a and the other is b, you can find the value of a and b:

You know that

a+b = 36

And

a/b = 1/2

You can use these equations to solve for a and b, or you can use the following simple method:

Find out how many units are in 1 part of the ratio. To do this, divide the total by the number of parts.

Number of parts: 1 + 2 = 3.

Number of units in each part: 36/3 = 12.

Then, multiply the number of units in each part by the number of parts in each variable.

a = 1 * 12 = 12

b = 2 * 12 = 24

As an aside . . .

Percentages are so frequently used that we should spend a little time on them here. The powerful thing about percentages is that they all have the same denominator: 100. When two fractions have the same denominator, comparisions can be made very easily.

0.21 = 21% = 21 / 100

You may be asked to solve some simple algebraic equations, mostly with ratios and proportions, and to convert between various units. Recall from Section 1: Ratios and Proportions, that a fraction is one way to write a ratio.

Let's say that there is a ratio that is expressed as a fraction, and you would like to know what that same ratio would be with a different denominator. This will be useful for comparing different ratios with one another and for presenting ratios in an easily understood manner.

Here is an example:

We'll start with a ratio of **2.1:10**

This can be written **2.1/10**

Now, for the moment, let's assume that this represents the number of disease cases in a population of 10 people. It can be awkward to think of fractions of people. You would never walk into a room and see 2.1 people standing there talking about the weather. For this reason, in Epidemiology, it is conventional to avoid fractions of people. To do this, you just increase the population size under consideration (the denominator). This is very easy to do if you increase the denominator size by a factor of ten. In that case, all you need to do is move the decimal one place to the right in both the numerator and the denominator (multiply the fraction by 10/10).

Like this:

2.1/10 = 21/100

You could keep going, if you wanted to,

2.1/10 = 21/100 = 210/1000 = 2100/10,000 = 21,000/100,000

Similarly, if you wanted to decrease the size of the denominator by a factor of ten for some reason, you would just move the decimal one place to the left in both the numerator and the denominator (multiply the fraction by 0.1/0.1).

Like this:

21/100 = 2.1/10 = 0.21/1 = 0.21

What if the ratio does not have a factor of ten as its denominator? For example, let's say we have a ratio of 8:20,000 and you need to know how many that is per 100,000.

8/20,000 = x/100,000

Here, we can't just move the decimal around. Happily, there is a simple method we can use to convert this fraction, and solve for x. Some people call it the 'flying x' method, or 'cross multiplication', because you multiply across the 'equals' sign in an 'x' pattern. To do this, first multiply the numerator of one fraction by the denominator of the other. This number goes on one side of the 'equals' sign.

Like this:

8/20,000 = x/100,000
8 * 100,000 = ?
800,000 = ?

Then, you do the same thing again with the remaining numerator and denominator. This number goes on the other side of the 'equals' sign.

Like this:

8/20,000 = x/100,000
8 * 100,000 = 20,000x
800,000 = 20,000x

All that is left to do is solve for 'x'. To do this in our example, we divide both sides by 20,000. So, we get

8/20,000 = x/100,000
8 * 100,000 = 20,000x
800,000 = 20,000x
x = 800,000/20,000
x = 40

so...

8/20,000 = 40/100,000

Let's look at two more examples:

8/21,463 = x/100,000
8 * 100,000 = 21,463x
21,463x = 800,000
x = 37.27

so...

8/21,463 = 37.27/100,000

Remember, if this were something like case/population size, we would want to avoid fractions of people, so we might write it like this:

3727/10,000,000

Here's the second example:

0.53/73 = x/100,000
73x = 0.53 * 100,000
73x = 53,000
x = 726

so...

0.53/73 = 726/100,000

This method works just as well if you want a different denominator. Here's an example:

1/8 = x/60
8x = 60
x = 7.5

so...

1/8 = 7.5/60

Frequently, you will have to consider units of measurement (like centimeters, people, or bushels) in your calculations. Sometimes, it will be important to have all parts of an equation in the same units. To do this, you may have to convert between units. For example, you may need to convert part of your equation from ounces to grams.

The easiest way to do this is to multiply the part of the equation that you need to convert by a fraction that is equal to one. Recall that a number divided by itself is equal to one, like 7/7, 21/21, or 50,000/50,000. Recall also that multiplying any part of an equation by one does not change its value at all.

What does all that mean for us here?

Just this:

If there are 5 ships in a flotilla, then

5 ships/1 flotilla = 1,and

1 flotilla/5 ships = 1

Here's a real example:

There are 0.035 ounces in a gram., so 1 g = 0.035 oz and...

1 g/0.035 oz = 1 = 0.035 oz/1 g

You can treat units like numbers. They can be multiplied or divided. This means they can also be cancelled. If you multiply two fractions, and they both contain the same units, except that one fraction has the unit in the denominator and the other in the numerator, they cancel each other out.

Like this:

oz/person * g/oz = g/person

The ounces cancel each other out.

So, let's say each person in a group got 8 ounces of steak, and we need to know how many grams each person got.

8 oz/1 person * 1g/0.035 oz = 8 g/0.035 people

This is a very awkward fraction, so we change the size of the denominator:

8/0.035 = x/1
0.035x = 8
x = 228.6

so...each person got 228.6 g of steak.

Let's try another example.

60 inches/1 measuring stick = x cm/measuring stick

We know from a table that 1 inch = 2.54 cm. So...

60 in/1 stick * 2.54 cm/1 in = 152.4 cm/1 stick = 152.4 cm/stick

This works just as well for denominators greater than 1, or for when you need to convert the denominator.

Here's an example with a denominator greater than 1:

1 forest = 40 trees

20 forest/45 glade = x trees/45 glade

20 forest/45 glade * 40 trees/1 forest = 800 trees/45 glade, and if you want...

800 trees/45 glade = 17.8 trees/glade or 178 trees/10 glade

Remember how to get that?

An example converting the units of the denominator:

40 dots/1 inch = x dots/cm
40 dots/1 inch * 1 inch/2.54 cm = 40 dots/2.54 cm = 15.75 dots/cm

This method of unit conversion is very helpful when setting up equations. If you do it this way, you can check to see if your equation is set up correctly by looking to see if you end up with the units you want.

Here is an example using made-up money conversions:

Let's say 2 pounds = 1 dollar, and 1 pound = 150 yen, but you need to know how much 20 dollars is in yen.

You could blithely start out with this equation:

20 dollars * 1 dollar/2 pounds * 1 pound/150 yen = x

Is this the right equation?

Let's check the units...

dollars * dollars/pounds * pounds/yen = $\textbf{dollars}^2$/yen

We get 'dollars squared over yen', not plain old yen. So, it can't be right. Let's try again using our cancellation method.

We want to go from dollars to yen via pounds. So, we start with dollars and convert to pounds.

dollars * pounds/dollars = pounds

Then we convert pounds to yen.

pounds * yen/pounds = yen

When we put this together, we get

dollars * pounds/dollars * yen/pounds = yen

This is what we want, so we just plug in the numbers and multiply it out.

20 dollars * 2 pounds/1 dollars * 150 yen/1 pound = 6000 yen/1 = 6000 yen

An exponent refers to the number of times a number is multiplied by itself. For example, 2 to the 3rd (written like this: 2^3) means:

2 x 2 x 2 = 8.

2^3 is not the same as 2 x 3 = 6.

Remember that a number raised to the power of 1 is itself. For example,

$a^1 = a$

$5^1 = 5$.

There are some special cases:

1. $a^0 = 1$

When an exponent is zero, as in 6^0, the expression is always equal to 1.

$a^0 = 1$

$6^0 = 1$

$14{,}356^0 = 1$

2. $a^{-m} = 1 / a^m$

When an exponent is a negative number, the result is always a fraction. Fractions consist of a numerator over a denominator. In this instance, the numerator is always 1. To find the denominator, pretend that the negative exponent is positive, and raise the number to that power, like this:

$a^{-m} = 1 / a^m$

$6^{-3} = 1 / 6^3$

You can have a variable to a given power, such as a^3, which would mean a x a x a. You can also have a number to a variable power, such as 2^m, which would mean 2 multiplied by itself m times. We will deal with that in a little while.

First let's look at how to work with variables to a given power, such as a^3.

There are five rules for working with exponents:

1. $a^m * a^n = a^{(m+n)}$

2. $(a * b)^n = a^n * b^n$

3. $(a^m)^n = a^{(m * n)}$

4. $a^m / a^n = a^{(m-n)}$

5. $(a/b)^n = a^n / b^n$

Let's look at each of these in detail.

1. $a^m * a^n = a^{(m+n)}$ says that when you take a number, a, multiplied by itself m times, and multiply that by the same number a multiplied by itself n times, it's the same as taking that number a and raising it to a power equal to the sum of m + n.

Here's an example where

a = 3
m = 4
n = 5

$a^m * a^n = a^{(m+n)}$

$3^4 * 3^5 = 3^{(4+5)} = 3^9 = 19,683$

2. $(a * b)^n = a^n * b^n$ says that when you multiply two numbers, and then multiply that product by itself n times, it's the same as multiplying the first number by itself n times and multiplying that by the second number multiplied by itself n times.

Let's work out an example where

a = 3
b = 6
n = 5

$(a * b)^n = a^n * b^n$

$(3 * 6)^5 = 3^5 * 6^5$

$18^5 = 3^5 * 6^5 = 243 * 7,776 = 1,889,568$

3. $(a^m)^n = a^{(m * n)}$ says that when you take a number, a , and multiply it by itself m times, then multiply that product by itself n times, it's the same as multiplying the number a by itself m * n times.

Let's work out an example where

a = 3
m = 4
n = 5

$(a^m)^n = a^{(m * n)}$

$(3^4)^5 = 3^{(4 * 5)} = 3^{20} = 3,486,784,401$

4. $a^m / a^n = a^{(m-n)}$ says that when you take a number, a, and multiply it by itself m times, then divide that product by a multiplied by itself n times, it's the same as a multiplied by itself m-n times.

Here's an example where

a = 3
m = 4
n = 5

$a^m / a^n = a^{(m-n)}$

$3^4 / 3^5 = 3^{(4-5)} = 3^{-1}$ **(Remember how to raise a number to a negative exponent.)**

$3^4 / 3^5 = 1 / 3^1 = 1/3$

5. $(a/b)^n = a^n / b^n$ says that when you divide a number, a by another number, b, and then multiply that quotient by itself n times, it is the same as multiplying the number by itself n times and then dividing that product by the number b multiplied by itself n times.

Let's work out an example where

a = 3
b = 6
n = 5

$(a/b)^n = a^n / b^n$

$(3/6)^5 = 3^5 / 6^5$

Remember 3/6 can be reduced to 1/2. So we have:

$(1/2)^5 = 243 / 7,776 = 0.03125$

Understanding exponents will prepare you to use logarithms.